

**On the motion of rotating bodies  
in field gravity theory and general relativity**

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**Abstract**

On the basis of Lagrangian formalism of relativistic field theory post-Newtonian equations of motion for a rotating body are derived in the frame of Feynman's quantum field gravity theory (FGT) and compared with corresponding geodesic equations in general relativity (GR). It is shown that in FGT the trajectory of a rotating test body does not depend on a choice of a coordinate system. The equation of translational motion of a gyroscope is applied to description of laboratory experiments with free falling rotating bodies and rotating bodies on a balance scale. Post-Newtonian relativistic effect of periodical modulation of the orbital motion of a rotating body is discussed for the case of planets of the solar system and for binary pulsars PSR B1913+16 and PSR B1259-63. In the case of binary pulsars with known spin orientations this effect gives a possibility to measure radiiuses of neutron stars.

## 1 Introduction

The discovery and continuous observations of binary pulsars has opened a new possibilities for testing relativistic gravity theories [1]. Timing observations of a pulsar in orbit with a stellar-mass companion require practical application of different relativistic gravity effects such as advance of periastron, gravitational redshift, time dilation, bending of light, Lense-Thirring precession, geodetic precession, and also gravitational radiation (see e.g. [2] - [9]).

All this effects are usually calculated in the frame of general relativity (GR) which is the geometrical approach to gravity. Observations of binary pulsars verified GR with high accuracy, however it is still important to apply other gravity theories for binary pulsars observations [5, 8].

Here it is considered the field gravity theory which elements have been formulated in 60-th by Feynman [10], Thirring [11], Kalman [12] and others (for recent discussion see Straumann [13], Baryshev [14]). In *Lectures on Gravitation* [10] Feynman gave a quantum relativistic field description of gravity similar to other fundamental forces (strong, weak, electromagnetic). Feynman's field gravity theory (FGT) is based on the quantum field theory and presents an alternative *non-geometrical* understanding the physics of gravitational interaction. He emphasized that though the theory of gravitation "has both a field and a geometrical interpretation", "the geometrical interpretation is not really necessary or essential to physics" ([10], p.113). According to Feynman, the gravity force between two masses is caused by the exchange of gravitons which are mediators of the gravitational interaction and actually represent the quanta of the relativistic tensor field  $\psi^{ik}$  in Minkowski space  $\eta^{ik}$ .

The fact that the field equations in FGT and GR is the same in linear approximation and that in FGT there is an iteration procedure which saves gauge-invariance and leads to the same non-linear field equations, is usually interpreted as complete identity of GR and FGT. However, after the iteration is fulfilled and geometrical interpretation is accepted, the new puzzling conceptual problems appear, among them the spacetime horizons and the break down of the energy-momentum conservation in geometrical approach (see review by Straumann [13]). Moreover the iteration procedure in the frame of FGT is

not uniquely defined operation because it requires to fix an expression for the energy momentum tensor of the gravitational field which is not uniquely defined in Lagrangian formalism and needs additional physical assumptions such as symmetry, positiveness of the field energy, traceless energy-momentum tensor for massless fields (see Landau & Lifshitz [15], Bogolubov & Shirkov [16], Baryshev [14]).

All classical PN relativistic gravity effects are the same in FGT and GR (see e.g. [11, 14, 20]). However here it is shown that there is a post-Newtonian effect which demonstrate a difference between FGT and GR and which may be tested by timing observations of binary pulsars and by future laboratory experiments with rotating masses, e.g. measuring a balance between differently rotating bodies. This post-Newtonian effect leads to periodic modulation of the orbital motion of a pulsar, caused by the velocity dependence of gravity force acting on a rotating body. In GR this effect is a coordinate-dependent concept and hence could not be uniquely defined, while in FGT there is definite coordinate-independent value for this relativistic gravity phenomenon. The effect was firstly discussed in [17] as a prediction of the field gravity theory for the Stanford gyroscope experiment [18]. Here this effect is discussed in connection with binary pulsars timing observations and formulation of the Galileo-like experiments with free falling rotating bodies or measuring a balance between rotating bodies.

In sec.2 post-Newtonian equations of motion of relativistic test particles in relativistic tensor gravity field are discussed and compared with geodesic equations of general relativity. In sec.3 equations of motion for a gyroscope orbiting around a central mass are derived and applied to the cases of laboratory experiments, planets of the solar system and binary pulsars as tests of the velocity dependence of the gravity force acting on rotating bodies. Conclusions are presented in sec.4.

## 2 Post-Newtonian equations of motion for test particles in field gravity theory and general relativity

## 2.1 Kalman's equation of motion in FGT

The basic principles of FGT are the same as for other relativistic quantum fields. These include the Minkowski space, the fundamental role of the inertial frames, the concepts of force and energy-momentum tensor, and also quantum uncertainty principle and the many-path approach.

Following [19] let us consider the motion of relativistic test particle with rest-mass  $m_0$ , 4-velocity  $u^i$ , 3-velocity  $\vec{v}$  in the gravitational field described by the symmetric tensor potential  $\psi^{ik}$  on flat Minkowski space-time where the Cartesian coordinates always exist and the metric tensor  $\eta^{ik} = \text{diag}(1, -1, -1, -1)$  (here we accept notations of the textbook [15]).

To derive the equation of motion in FGT one may use the stationary action principle in the form of the sum of the free particles and the interaction parts:

$$\delta S = \delta\left(\frac{1}{c} \int (\Lambda_{(p)} + \Lambda_{(int)}) d\Omega\right) = 0 \quad (1)$$

where  $d\Omega$  is the element of 4-volume and the variation of the action is made with respect to the particle trajectories  $\delta x^i$  for *fixed* gravitational potential  $\psi^{ik}$ .

The free particle Lagrangian is

$$\Lambda_{(p)} = -\eta_{ik} T_{(p)}^{ik} \quad (2)$$

and the interaction Lagrangian is

$$\Lambda_{(int)} = -\frac{1}{c^2} \psi_{ik} T_{(p)}^{ik} \quad (3)$$

where the energy-momentum tensor (EMT) of the point particle is

$$T_{(p)}^{ik} = m_0 c^2 \delta(\vec{r} - \vec{r}_p) \left\{ 1 - \frac{v^2}{c^2} \right\}^{1/2} u^i u^k \quad (4)$$

Inserting equations (4),(3) into equation(1) and taking into account that  $ds^2 = dx_l dx^l$  we get

$$\int (m_0 c \delta(\sqrt{dx_l dx^l}) + \frac{m_0}{c} \delta(\psi_{ik} \frac{dx^i dx^k}{\sqrt{dx_l dx^l}})) = 0 \quad (5)$$

Opening the variation and integrating by parts, and taking into account that the variation is made for the fixed values of the integration limits, we find

$$\int m_0 c du_i \delta dx^i + \frac{2m_0}{c} d(u^k \psi_{ik}) \delta x^i - \frac{m_0}{c} d(\psi_{lk} u^l u^k u_i) \delta x^i - \frac{m_0}{c} u^k \delta \psi_{ik} dx^i = 0 \quad (6)$$

Consider also that

$$du^i = \frac{du^i}{ds} ds; \quad dx^i = u^i ds; \quad \delta\psi_{ki} = \psi_{ki,l} \delta x^l;$$

$$d(u^l u^k u_i \psi_{lk}) = u^l u_i u^k \psi_{lk,n} dx^n + u^l u^k \psi_{lk} du_i + 2\psi_{lk} u^l u_i du^k;$$

finally we get the following equation of motion for test particles in the field gravity theory [19]:

$$A_k^i \frac{dp^k}{ds} = -m_0 B_{kl}^i u^k u^l \quad (7)$$

where  $p^k = m_0 c u^k$  is the 4-momentum of the test particle,  $(\cdot)_{,i} = \frac{\partial(\cdot)}{\partial x^i}$ , and

$$A_k^i = (1 - \frac{1}{c^2} \psi_{nl} u^n u^l) \eta_k^i - \frac{2}{c^2} \psi_{kn} u^n u^i + \frac{2}{c^2} \psi_k^i \quad (8)$$

$$B_{kl}^i = \frac{2}{c^2} \psi_{k,l}^i - \frac{1}{c^2} \psi_{kl}^{,i} - \frac{1}{c^2} \psi_{kl,n} u^n u^i \quad (9)$$

The rest mass of the test particle can be canceled, hence in the field gravity theory  $m_{in} = m_g = m_0$  without initial equivalence postulate as was emphasized by Thirring [11].

The equation (7) is identical to the equation of motion derived by Kalman [12] in another way, by considering the relativistic Lagrange function  $L$  defined as  $S = \int L \frac{ds}{c}$  and relativistic Euler equation:

$$\frac{d}{ds} \left( \frac{\partial L}{\partial u^k} u^k - L \right) u_i + \frac{\partial L}{\partial u^i} = -\frac{\partial L}{\partial x^i} \quad (10)$$

Inserting in equation (10) the following expression of the relativistic Lagrange function

$$L = -m_0 c^2 - m_0 \psi_{ik} \frac{dx^i}{ds} \frac{dx^k}{ds} \quad (11)$$

one gets Kalman's equations of motion [12], which may be transformed into equation (7).

## 2.2 PN equations of motion in FGT

In post-Newtonian approximation we keep terms an order of  $v^2/c^2 \sim \varphi_N/c^2 \ll 1$  in equation (7). For PN accuracy we need calculations of the  $\psi^{00}$  component with the same order, while other components of the tensor gravitational potential  $\psi^{ik}$  can be

calculated in Newtonian approximation. Under these assumptions from equation (7) for ( $i = \alpha$ ) we get the expression for the PN 3-dimensional gravity force (which we shall call the Poincaré gravity force remembering his pioneer work in 1905 on relativistic gravity force in flat space-time):

$$\begin{aligned}\vec{F}_{Poincare} &= \frac{d\vec{p}}{dt} = -m_0\left\{(1 + \frac{3}{2}\frac{v^2}{c^2} + 3\frac{\phi}{c^2})\vec{\nabla}\phi - 3\frac{\vec{v}}{c}(\frac{\vec{v}}{c} \cdot \vec{\nabla}\phi)\right\} \\ &\quad - m_0\left\{3\frac{\vec{v}}{c}\frac{\partial\phi}{c\partial t} - 2\frac{\partial\vec{\Psi}}{c\partial t} + 2(\frac{\vec{v}}{c} \times \text{rot}\vec{\Psi})\right\}\end{aligned}\quad (12)$$

where  $\phi = \psi^{00}$ ,  $\vec{\Psi} = \psi^{0\alpha} = -\psi_{0\alpha}$ .

An important advance of FGT is that in Minkowski space according to Noether's theorem there is well-defined concept of the energy-momentum tensor and its conservation in the form which is usual for relativistic quantum fields. Hence the energy of gravity field is localized positive physical quantity and is given by the 00-component of the energy-momentum tensor (and not pseudo-tensor as in GR). For post-Newtonian applications it is sufficient to use the weak field approximation for the calculation of the gravity field EMT, which gives in the case of a static spherically symmetric massive body the following value for the 00-component ([11, 20]):

$$T_{(g)}^{00} = \varepsilon_{(g)} = \frac{1}{8\pi G}(\vec{\nabla}\varphi_N)^2 \quad \frac{\text{ergs}}{\text{cm}^3} \quad (13)$$

where  $\varphi_N$  is Newtonian potential. The energy of the gravity field gives the following PN correction to the 00-component of tensor gravitational potential ([11, 20]) :

$$\phi = \psi^{00} = \varphi_N + \frac{1}{2}\frac{\varphi_N^2}{c^2} \quad (14)$$

Hence in the frame of FGT the energy of the gravity field is observable quantity which may be measured by observations of test particles motion. Take into account the expression (14) for the 00-component of the gravitational potential, we get corresponding PN 3-acceleration of the test particle :

$$\begin{aligned}\frac{d\vec{v}}{dt} &= -(1 + \frac{v^2}{c^2} + 4\frac{\varphi_N}{c^2})\vec{\nabla}\varphi_N + 4\frac{\vec{v}}{c}(\frac{\vec{v}}{c} \cdot \vec{\nabla}\varphi_N) \\ &\quad + 3\frac{\vec{v}}{c}\frac{\partial\varphi_N}{c\partial t} - 2\frac{\partial\vec{\Psi}}{c\partial t} + 2(\frac{\vec{v}}{c} \times \text{rot}\vec{\Psi})\end{aligned}\quad (15)$$

From the ( $i = 0$ ) component of equation (7) it follows the expression for the work of the Poincaré force:

$$\frac{dE_k}{dt} = \vec{v} \cdot \vec{F}_{Poincare} = -m_0 \vec{v} \cdot \left\{ \left( 1 - \frac{3}{2} \frac{v^2}{c^2} + 3 \frac{\phi}{c^2} \right) \vec{\nabla} \phi - 3 \frac{\vec{v}}{c} \frac{\partial \phi}{\partial t} + 2 \frac{\partial \vec{\Psi}}{\partial t} \right\} \quad (16)$$

An important particular case is the static spherically symmetric weak gravitational field for which  $\vec{\Psi} = 0$ ,  $\partial \phi / \partial t = 0$ ,  $\psi^{ik} = \text{diag}(\phi, \varphi_N, \varphi_N, \varphi_N)$  hence PN 3-acceleration will have the simple form:

$$\left( \frac{d\vec{v}}{dt} \right)_{FGT} = -\left( 1 + \frac{v^2}{c^2} + 4 \frac{\varphi_N}{c^2} \right) \vec{\nabla} \varphi_N + 4 \frac{\vec{v}}{c} \left( \frac{\vec{v}}{c} \cdot \vec{\nabla} \varphi_N \right) \quad (17)$$

From equation of motion (17) it is clear that the acceleration of a test particle depends on the value and the direction of its velocity and this is coordinate independent relativistic gravity effect.

For circular motion  $\vec{v} \perp \vec{\nabla} \varphi_N$  hence PN 3-acceleration is

$$\left( \frac{d\vec{v}}{dt} \right)_{FGT} = -\left( 1 + \frac{v^2}{c^2} + 4 \frac{\varphi_N}{c^2} \right) \vec{\nabla} \varphi_N \quad (18)$$

For radial motion  $\vec{v} \uparrow \downarrow \vec{\nabla} \varphi_N$  the 3-acceleration is

$$\left( \frac{d\vec{v}}{dt} \right)_{FGT} = -\left( 1 - 3 \frac{v^2}{c^2} + 4 \frac{\varphi_N}{c^2} \right) \vec{\nabla} \varphi_N \quad (19)$$

### 2.3 PN equations of motion in GR

In GR the equation of motion of a test particle is the geodesic equation [15] :

$$\frac{du^i}{ds} = -\Gamma_{kl}^i u^k u^l \quad (20)$$

where  $u^i$  is 4-velocity of the test particle and  $\Gamma_{kl}^i$  is the Christoffel symbols.

Post-Newtonian geodesic equation has been carefully studied in relativistic celestial mechanics and according to [21, 22] PN 3-acceleration of a test particle in the case of static spherically symmetric gravitational field is:

$$\begin{aligned} \left( \frac{d\vec{v}}{dt} \right)_{GR} = & -\left\{ 1 + (1 + \alpha) \frac{v^2}{c^2} + (4 - 2\alpha) \frac{\varphi_N}{c^2} - 3\alpha \left( \frac{\vec{r}}{r} \cdot \frac{\vec{v}}{c} \right)^2 \right\} \vec{\nabla} \varphi_N \\ & + (4 - 2\alpha) \frac{\vec{v}}{c} \left( \frac{\vec{v}}{c} \cdot \vec{\nabla} \varphi_N \right) \end{aligned} \quad (21)$$

where  $\vec{v} = d\vec{r}/dt$ ,  $\varphi_N = -GM/r$ ,  $\vec{\nabla}\varphi_N = GM\vec{r}/r^3$ .

The very important quantity in equation(21) is the parameter of coordinate system  $\alpha$  which depend on the choice of particular coordinate system and for example has the following values:  $\alpha = 2$  for Painlevé coordinates,  $\alpha = 1$  for Schwarzschild coordinates,  $\alpha = 0$  for harmonic or isotropic coordinates. Hence the trajectory of a test particle is a coordinate-dependent concept, i.e. it depends on the choice of a coordinate system due to parameter  $\alpha$ .

In the case of circular motion the 3-acceleration is:

$$\left(\frac{d\vec{v}}{dt}\right)_{GR} = -\left\{1 + (1 + \alpha)\frac{v^2}{c^2} + (4 - 2\alpha)\frac{\varphi_N}{c^2}\right\}\vec{\nabla}\varphi_N \quad (22)$$

For radial motion the 3-acceleration is:

$$\left(\frac{d\vec{v}}{dt}\right)_{GR} = -\left\{1 - 3\frac{v^2}{c^2} + (4 - 2\alpha)\frac{\varphi_N}{c^2}\right\}\vec{\nabla}\varphi_N \quad (23)$$

According to [21] in GR there are "coordinate-dependent (unmeasurable) quantities" and "measurable quantities" which may be directly obtainable from observation without involving any theoretical data (such as the laws of light propagation). For example the radius  $r$  of the circular orbit is a coordinate-dependent, unmeasurable quantity, while the masses  $M$  and  $m$  do not depend on coordinate system and hence are measurable quantities.

The geodesic equations (21,22, 23) are identical with equations (17,18,19) of FGT only for harmonic or isotropic coordinates where  $\alpha = 0$ . Fock [23] also emphasized the special role of harmonic coordinates in GR.

### 3 Translational motion of a rotating body in FGT

#### 3.1 PN equations of motion of a gyroscope in FGT

Let us consider Poincaré gravity force acting on a rotating body. From equation (12) in the case of a gyroscope motion in static spherically symmetric gravitational field it follows the expression for the elementary gravity force  $dF_P$  acting on each elementary mass  $dm$  of the gyroscope:

$$d\vec{F}_P = -\left\{(1 + \frac{3}{2}\frac{v^2}{c^2} + 4\frac{\varphi_N}{c^2})\vec{\nabla}\varphi_N - 3\frac{\vec{v}}{c}(\frac{\vec{v}}{c} \cdot \vec{\nabla}\varphi_N)\right\}dm \quad (24)$$

For a rotating solid body the total gravity force is the sum of elementary forces acting on elementary masses:

$$\vec{F}_P = \int d\vec{F}_P \quad (25)$$

Taking into account that the velocity  $v$  of an element  $dm$  may be presented in the form

$$v = \vec{V} + [\vec{\omega}\vec{r}] \quad (26)$$

where  $\vec{V}$  is the translational velocity of the body,  $\vec{\omega}$  is the angular velocity,  $\vec{r}$  is the radius vector of an element  $dm$  relative to its center of inertia, so that  $\int \vec{r} dm = 0$ .

Inserting (26) into (24) and (25) we get

$$\begin{aligned} \vec{F}_P = & -M\{(1 + \frac{3}{2}\frac{V^2}{c^2} + 4\frac{\varphi_N}{c^2} + \frac{3}{2}\frac{I\omega^2}{Mc^2})\vec{\nabla}\varphi_N \\ & - 3\frac{\vec{V}}{c}(\frac{\vec{V}}{c} \cdot \vec{\nabla}\varphi_N) - \frac{3}{Mc^2} \int [\vec{\omega}\vec{r}]([\vec{\omega}\vec{r}] \cdot \vec{\nabla}\varphi_N) dm\} \end{aligned} \quad (27)$$

where  $M = \int dm$  is the total mass of the body,  $I$  is its moment of inertia.

Under the gravity force (27) the rotating body will get the 3-acceleration according to general relation [15] :

$$\frac{dv}{dt} = \frac{\sqrt{1-v^2/c^2}}{m_0}(\vec{F} - \frac{\vec{v}}{c}(\frac{\vec{v}}{c} \cdot \vec{F})) \quad (28)$$

which may be written in the form:

$$\begin{aligned} \frac{d\vec{V}}{dt} = & -(1 + \frac{V^2}{c^2} + 4\frac{\varphi_N}{c^2} + \frac{3}{2}\frac{I\omega^2}{Mc^2})\vec{\nabla}\varphi_N \\ & + 4\frac{\vec{V}}{c}(\frac{\vec{V}}{c} \cdot \vec{\nabla}\varphi_N) + \frac{3}{Mc^2} \int [\vec{\omega}\vec{r}]([\vec{\omega}\vec{r}] \cdot \vec{\nabla}\varphi_N) dm \end{aligned} \quad (29)$$

Equation of motion of a rotating body (29) shows that the orbital velocity of the center of mass of the body will have additional perturbations due to rotation of the body. The last term in (29) depends on direction and value of the angular velocity  $\vec{\omega}$ . It has an order of magnitude  $v_{rot}^2/c^2$  and may be measured in laboratory experiments and astrophysical observations.

### 3.2 Laboratory experiments with rotating bodies

The most straightforward application of equation (29) is to perform "Galileo-2000" experiment (which is an improved version of famous Stevinus-Grotius-Galileo experiment

with free falling bodies in the Earth gravity field) just taking into account rotation of the bodies.

Indeed let us consider three balls on the top of a tower (like the 110-m Drop Tower of the Bremen University). The first ball is non-rotating and according to equation (29) its free fall acceleration is:

$$\vec{g}_1 = \left( \frac{d\vec{V}}{dt} \right)_1 = -\left( 1 - 3\frac{V^2}{c^2} + 4\frac{\varphi_N}{c^2} \right) \vec{\nabla} \varphi_N \quad (30)$$

Let the rotation axis of the second ball be parallel to the gravity force, i.e.  $\vec{\omega} \parallel \vec{\nabla} \varphi_N$ , hence its free fall acceleration is:

$$\vec{g}_2 = \vec{g}_1 \left( 1 + \frac{3}{5} \frac{R^2 \omega^2}{c^2} \right) \quad (31)$$

where it is taken into account that for homogeneous ball with radius  $R$  and mass  $M$  the moment of inertia is  $I = \frac{2}{5}MR^2$ .

Let the rotation axis of the third ball be orthogonal to the gravity force, i.e.  $\vec{\omega} \perp \vec{\nabla} \varphi_N$ , hence its free fall acceleration is:

$$\vec{g}_3 = \vec{g}_1 \left( 1 - \frac{3}{10} \frac{R^2 \omega^2}{c^2} \right) \quad (32)$$

From equations (30),(31),(32) it follows that considered three balls will reach the ground at different moments. True the difference is very small, for example if the radius of the ball is  $R = 10$  cm and its angular velocity  $\omega = 10^3$  rad/sec, then the expected difference in falling time from 110 m tower will be  $\Delta t \approx (1/2)(\Delta g/g)t \approx 2.5 \times 10^{-13}$  sec.

For the NASA's gravity probe B (GPB) experiment [18] the radius of the gyroscope is  $R \approx 2$  cm and it rotates with about 2000 revolutions per second. The expected effect of the periodical perturbation of the acceleration of the gyroscope center of mass relative to non-rotating satellite center of mass is about  $\delta g/g \approx 6 \times 10^{-13}$  which is in principle measurable [17].

Another type of laboratory experiment for direct testing the velocity dependence of the Poincaré gravity force (27) is to weigh a rotating bodies. If two bodies are at the balance and at a moment they start to rotate with different orientations of the rotation axes then the balance will be violated and hence measured by a scale. The expected difference in forces is again about  $\omega^2 R^2/c^2$ .

In these laboratory experiments there are no problem with choosing a coordinate system at all. The height of a tower and the moments of the contact of the rotating bodies with the ground, and also readings of a balance scales are directly measurable quantities. Equation of motion (17) in field gravity theory gives the uniquely defined value for these laboratory experiments, while geodesic equation (21) includes arbitrary parameter  $\alpha$  of a coordinate system.

### 3.3 Periodical modulation of planet's orbits

Post-Newtonian celestial mechanics in FGT does not depend on a choice of coordinate systems and is identical with PN approximation of GR only in harmonic or isotropic coordinates. This is why all classical post-Newtonian relativistic gravity effects are the same in FGT and GR. Note that according to [24] "astronomers have adopted the fairly standard convention of using "isotropic coordinates" rather than "Schwarzschild coordinates" when analyzing the solar system" ([24],p.1097).

According to [21] in GR the effects which depend on the choice of coordinate system (e.g. the parameter  $\alpha$ ) are unmeasurable. However there are such effects which are coordinate dependent in GR but are coordinate independent in FGT. An example of such effect is additional perturbation terms in equations of motion of rotating bodies which are determined by angular velocities of the bodies (see equation (29)).

For a planet rotating with the angular velocity  $\vec{\omega}$  in orbit with semi-major axis  $a$ , and orbital angular momentum  $\vec{L}$  the effect of rotation leads to periodical modulation of the orbital radius. The amplitude of the radius perturbation  $\Delta a$  may be roughly estimated as:

$$\Delta a \approx a \frac{\omega^2 R^2}{c^2} \sin^2 \theta \quad (33)$$

where  $R$  is the radius of planet,  $\theta$  is the angle between  $\vec{\omega}$  and  $\vec{L}$ .

According to equation (33) the amplitude of the orbital modulation is about 6 cm for the Earth, 1 cm for Mars, 4.2 m for Jupiter and 340 m for Saturn. Future high accuracy radar observations of the solar system will test this tiny effects.

### 3.4 Rapidly rotating stars and pulsars in binary systems

In a binary stellar system the rapidly rotating companion will produce the relativistic effect of the periodical modulation of the orbital motion. However for usual stars there are many other non-relativistic effects which usually hide small relativistic effects.

The best candidates for testing relativistic gravity effects are pulsars in binary systems. Timing observations of pulsars, i.e. the measurements of the time of arrival of pulsar signals at a radio telescope, are the high-precision experiments in astronomy. About 10% of the known pulsars are members of binary star systems, i.e. in orbit around a white dwarf, neutron star, or main-sequence star companion. Timing observations of these binary pulsars may be used for testing the periodical modulation of the orbit of the rotating bodies.

In the case of equal masses of stars it is needed to generalize the above test-body approach to the case of two-body problem. However for rough estimation of the expected effect it is enough to use equation (29). According to (33) the amplitude of the modulation of the orbit are determined by the spin angular velocity  $\omega$ , the radius  $R$  of the pulsar (or more exactly, the distribution of the mass and velocity in the pulsar, i.e. its moment of inertia  $I_{\alpha,\beta}$  and spin angular momentum  $\vec{S}$ ), semi-major axis  $a$  and the angle  $\theta_p$  between pulsar's spin and direction of orbital angular momentum.

Because the amplitude of the modulation effect is proportional to the semi-major axis, the largest effect may be observed for the special class of the long-orbital period binary pulsars [25]. For known  $\omega = 2\pi/P_p$ ,  $a$  and  $R$  it is possible to determine the angle  $\theta_p$ . If for a pulsar there is sufficiently accurate estimation of the angle  $\theta_p$ , then it is possible to estimate the radius of the pulsar  $R$ .

Let us make estimation for the expected effect in the case of two well-studied binary pulsars PSR B1913+16 and PSR B1259-63. The amplitude of the orbital modulation is given by (33) and the value of the expected variation in arrival times of pulses  $\Delta\tau = \Delta a/c$  may be written in the form:

$$\Delta\tau \approx 1.75 \cdot 10^{-5} (\text{sec}) \left( \frac{x}{1 \text{ s}} \right) \left( \frac{50 \text{ ms}}{P_p} \right)^2 \left( \frac{R}{10 \text{ km}} \right)^2 \sin^2 \theta_p \quad (34)$$

where  $x = a \cdot \sin i/c$  is the projected semi-major axis,  $i$  is the inclination angle of the

orbit,  $P_p$  is the period of the pulsar,  $R$  is the radius of the neutron star. and  $\theta_p$  is the misalignment angle of the pulsar spin to the orbital momentum vector.

For PSR B1913+16: period  $P_p = 59$  ms,  $x = 2.34$  s, and  $\theta_p \approx 22^\circ$  [7], hence the expected periodical variations in arrival time due to rotational effect is about  $6\ \mu\text{s}$ . For PSR B1259-63 we have  $P_p = 47.8$  ms,  $x = 1296$  s [25] and for  $\theta = 10^\circ$  the time variations will be about 0.7 ms. These numbers are close to the time residuals existing for these pulsars and it shows possibility to test the discussed effect.

## 4 Conclusions

Using Lagrangian formalism of relativistic field theory it is derived PN equations of motion of a rotating body in the framework of Feynman's field gravity theory.

The trajectory of a test particle does not depend on a choice of a coordinate system and this is an essential property of field gravity theory which allow to make uniquely defined predictions for terrestrial laboratory experiments with rotating masses in Earth's gravity field. In these laboratory experiments there are no problem with choosing a coordinate system at all. The height of a tower and the moments of the contact of the rotating bodies with the ground, and also readings of a balance scales are directly measurable quantities. Equation of motion (17) in field gravity theory gives the uniquely defined value for these laboratory experiments, while geodesic equation (21) includes arbitrary parameter  $\alpha$  of a coordinate system.

In the case of astronomical observations of distant objects it is important to consider also effects of emission, propagation and detection of photons, however it does not change the concept of coordinate independence of the trajectory of a test particle.

Post-Newtonian periodical modulation of planetary motion in the solar system may be measured by future radar observations. Timing observations of pulsars in binary systems is the best test of this relativistic gravity effect. The expected arrival time variation due to pulsar rotation lies in interval from  $1\ \mu\text{s}$  up to  $1\ \text{ms}$  depending mainly on the value of the semi-major axis of pulsar orbit, misalingnment angle of pulsar spin, angular velocity of its rotation and the radius of a neutron star. In the case of binary

pulsars with known spin orientations this effect gives a possibility to measure radiiuses of neutron stars.

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